University of Bahrain
College of Information Technology
Department of Computer Science
ITCS251/252 Discrete Mathematics
Second Semester 2011/2012

First Exam — 1 hour.

STUDENT NAME	
STUDENT#	
SECTION	

QUESTION#	MARKS		REMARKS
1	7	7	
2	~~8~~ 6	6	
3	6	5	
4	6	6	
5	8	8	
TOTAL	35	32	

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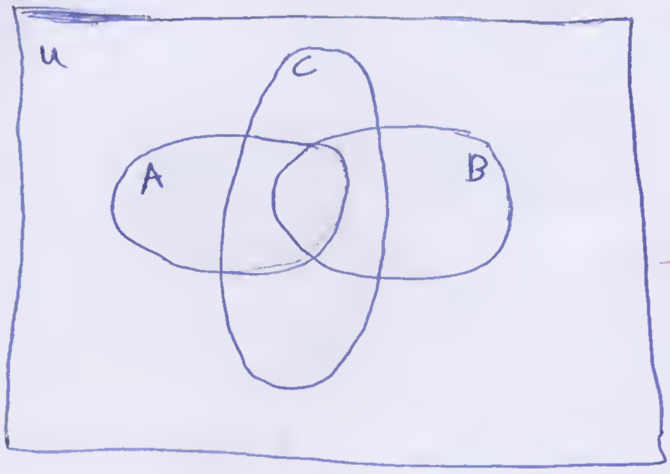
Q1. (a) [3 points] Draw a Venn diagram showing that $A \cap B \subset A \cap C$ but $B \not\subseteq C$.



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$A_1 = \{x \in Z \mid x < -1 \text{ or } x > 1\} = \{\dots, -3, -2, 2, 3, \dots\}$

(b) [2 points × 2] Let $A_i = \{x \in \mathbf{Z} \mid x < -i \text{ or } x > i\}$. Find $A_2 = \{x \in Z \mid x < -2 \text{ or } x > 2\} = \{\dots, -4, -3, 3, 4, \dots\}$

$\overline{A_1} - \{0\} = \{-1, 0, 1\} - \{0\} = \{-1, 1\}$ 1 and $|\overline{A_2}| = |\{-2, -1, 0, 1, 2\}| = 5$ 1

$\mathcal{P}(\overline{A_1}) = P(\{-1, 0, 1\}) = \{\phi, \{-1, 0, 1\}, \{-1\}, \{0\}, \{1\}, \{-1, 0\}, \{-1, 1\}, \{0, 1\}\}$ 2

Q2. (a) Consider the following proposition:

If we are not in vacation, then we go to the university and study logic.

(1) [2 points] Write the proposition in symbolic form using only $\neg, \wedge, \vee$.

Let p: we are not in vacation, q: we go to the university, r: study logic

$p \longrightarrow (q \wedge r)$

in symbolic: $\equiv \sim p \vee (q \wedge r)$ 2

(2) [2 points] Write the negation of the proposition in symbolic form.

$\boxed{6}$

$\sim(\sim p \vee (q \wedge r)) \equiv p \wedge \sim(q \wedge r)$

$\equiv p \wedge (\sim q \vee \sim r)$ 2

(3) [2 points] Write the contrapositive of the proposition in English. $\sim(q \wedge r) \to \sim p \equiv (\sim q \vee \sim r) \to \sim p$

If we don't go to the university or we don't study logic, then we are in vacation. 2

(b) [2 points] Write the following proposition using "*necessary condition*".

~~*It is not the case that*~~ *if Hasan plays football and basketball, then he is not in his office or he is at home.*

Hasan not in his office or he is at home are necessary conditions for Hasan to play football and basketball.

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Q3. [6 points] Write the truth table for the following proposition.

$$(q \vee \neg r) \leftrightarrow (\neg r \rightarrow \neg p)$$

p	q	r	$(q \vee \sim r)$	$(\sim r \rightarrow \sim p)$	$(q \vee \sim r) \leftrightarrow (\sim r \rightarrow \sim p)$
T	T	T	T	F → F = T	T
T	T	F	T	T → F = F	F
T	F	T	F	F → T = T	F
T	F	F	T	T → ~~T~~ F = ~~T~~ F	~~T~~ F
F	T	T	T	F → F = T	T
F	T	F	T	T → ~~F~~ T = ~~F~~ T	F T
F	F	T	F	F → T = T	F
F	F	F	T	T → T = T	T
			2pts	1.5	1.5

Q4. Show whether the following propositions are true or false. Justify your answer without using truth tables.

(1) [2 points] Let r be always true, $p \vee q \equiv p \vee q \wedge r$.

if r always true

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$\because p \vee q \wedge r \equiv p \vee q \wedge t \equiv p \vee q$

$\therefore p \vee q \equiv p \vee q \wedge r$

(2) [2 points] $\neg p \rightarrow \neg q$ is logically equivalent to $p \rightarrow q$.

$\sim p \rightarrow \sim q \equiv p \vee \sim q$

$p \rightarrow q \equiv \sim p \vee q$

2

$\because p \vee \sim q \not\equiv \sim p \vee q$

$\therefore \sim p \rightarrow \sim q$ is not logically equivalent to $p \rightarrow q$

(3) [2 points] $(p \wedge q) \vee \neg p \vee \neg q \equiv T$, where T is a tautology.

$\sim p \vee \sim q \equiv \sim(p \wedge q)$

$(p \wedge q) \vee \sim(p \wedge q)$

2

$T \vee F \equiv T$

$F \vee T \equiv T$

$\therefore (p \wedge q) \vee \sim p \vee \sim q \equiv T$

Q5. [8 points] Use inference rules to prove that the following argument is valid.

$$\begin{array}{ll} ① & p \to \neg q \\ ② & \neg r \wedge \neg s \equiv \sim(r \vee s) \\ ③ & \neg q \to (r \vee s) \\ \hline & \therefore \neg p \vee t \end{array}$$

from (1) and (3)

$p \to \sim q$

$\sim q \to (r \vee s)$

④ $\therefore p \to (r \vee s)$ ✓

From (4) and (2)

$p \to (r \vee s)$ ✓

$\sim(r \vee s)$

⑤ $\therefore \sim p$

From (5):

$\sim p$ ✓

$\therefore \sim p \vee t$

$\therefore$ the argument is valid.

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